STOCHASTIC REACCELERATION OF COSMIC RAYS IN THE INTERSTELLAR MEDIUM

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ABSTRACT

The effects of reacceleration on cosmic rays have been studied over a wide charge and energy range using a model of reacceleration by the interstellar turbulence. We take into account only inevitable stochastic reacceleration of cosmic rays by the random hydrodynamic waves, which supposedly exist in the interstellar medium and provide a means for cosmic-ray scattering and spatial diffusion in the Galaxy. Our calculations reproduce not only the B/C ratio but also the H and He data over the entire energy range where the measurements are available. However, the sub-Fe to Fe ratio is not fitted as well as the B/C ratio, and the reacceleration effect does not seem to remove the need for truncation of short path lengths, which is apparently required by the standard leaky box model. This work demonstrates that the cosmic-ray data can be represented at least as well by a reacceleration model with a simple rigidity power-law escape length, which agrees with the Kolmogorov-type spectrum of hydromagnetic turbulence, as they can by the standard leaky box model with its ad hoc escape lengths.

Subject headings: acceleration of particles — cosmic rays — MHD — turbulence — waves

1. INTRODUCTION

The acceleration of cosmic rays over the whole Galaxy, accompanied by nuclear fragmentation and production of secondary nuclei, cannot serve as the main mechanism for accelerating particles with energies $E \gtrsim 1$ GeV nucleon$^{-1}$. In this strong continuous acceleration case, particles of higher energy should spend a longer time in the system, which would result in an increase in the abundance of secondary nuclei as energy increases (Hayakawa 1969; Cowik 1986; Cesarsky 1987; Ptuskin 1991). However, observations show that both the abundance of secondary nuclei and the corresponding escape length $X_e$ (the mean thickness of matter traversed by cosmic rays) decrease with energy for $E \gtrsim 1$ GeV nucleon$^{-1}$; see, for example, Webber (1993). Hence significant continuous interstellar acceleration is excluded for these energies. On the other hand, strong acceleration could operate at low energies where the secondary abundances increase with energy.

Surprising at first glance was the result of numerical calculations by Simon, Heinrich, & Mattics (1986), which was subsequently confirmed in several reports (Wandel et al. 1987; Ferrando & Soutoul 1987; Osborne & Ptuskin 1988; Giler et al. 1989; Letaw, Silverberg, & Tsao 1993; Heinbach & Simon 1993). Specifically, it was shown that weak reacceleration of cosmic rays can provide the observed energy dependence of secondary abundances, even with a weaker decrease of the escape length with energy as compared to the case with an absence of any reacceleration. The qualitative difference in the behavior of cosmic-ray spectra in two limiting cases of strong and weak reacceleration are illustrated in Figure 1; see the discussions by Cesarsky (1987) and Ptuskin (1991).

In view of significant differences among the models of different authors, we shall not analyze their results in detail, but some common features will be outlined. The advantage of all models with reacceleration is a weak energy dependence of cosmic-ray anisotropy, evidently compatible with the measurements around $10^{12}-10^{14}$ eV. The anisotropy is caused by the cosmic-ray leakage from the Galaxy. The models also naturally reproduce the observed ratio of secondary to primary nuclei with a single-power-law dependence of escape length on particle rigidity. This is in contrast with the empirical models without reacceleration, where the escape length must be flattened for energies $\gtrsim 1$ GeV nucleon$^{-1}$. We do not have a definitive explanation for this flattening, but suggested explanations in the models without reacceleration include the action of a Galactic wind and turbulent diffusion (Jones 1979; Chuvil'gin & Ptuskin 1993; Ptuskin & Zirakashvili 1993).

However, it is still not clear whether reacceleration is compatible with the data at low energies where the efficiency of reacceleration should become stronger. In their early work, Silberberg et al. (1983) pointed out that a regular energy boost for low-energy cosmic rays could explain the observed data. Meyer (1985) discussed this problem in more detail. On the other hand, Stephens & Golden (1990) came to the conclusion that the H and He data below 5 GeV nucleon$^{-1}$ are not reproduced in the models without reacceleration (see also Heinbach & Simon 1993).

In the present paper we study the “minimal” model of reacceleration by the interstellar turbulence considered by Osborne & Ptuskin (1988). The term “minimal” means that we take into account only inevitable stochastic reacceleration of cosmic rays by the turbulence, which supposedly exists in the interstellar medium and provides a means for cosmic-ray scattering and confinement in the Galaxy. The efficiency of scattering can be obtained from the data on secondary nuclei. The asymptotic analytical solution for this model at energies $E \gtrsim 1$ GeV nucleon$^{-1}$ was given by Osborne & Ptuskin (1988). A numerical study with a similar model, but in the energy range which covered low energies, was recently carried out by Heinbach & Simon (1993). Here we develop this approach further and investigate transport and nuclei fragmentation of cosmic rays over a wide charge and energy range.
2. TRANSPORT EQUATION

We consider propagation of cosmic rays in the interstellar medium with random hydromagnetic waves. The steady state transport equation for the isotropic part of the momentum distribution function of cosmic rays in the one-dimensional case has the following form (see Berezinskii et al. 1990):

\[
\frac{\partial}{\partial z} D_j \frac{\partial f_j}{\partial z} + \frac{\rho}{m} v \sigma f_j - \frac{1}{p^2} \frac{\partial}{\partial p} p^2 K_j \frac{\partial f_j}{\partial p} + \frac{1}{p^2} \frac{\partial}{\partial p} \left[ p^2 \left( \frac{dp}{dt} \right)_{j, \text{ion}} f_j \right] = \frac{d_j}{E} + \sum_{k < j} S_{jk}.
\]  

(1)

Here the distribution function \( f_j(z, p) \) is normalized as \( N = \int dp \, p^2 f_j \), where \( N \) is the cosmic-ray number density; the subscript \( j = 1, 2, 3, \ldots \) characterizes the type of nucleus starting with the heaviest one (no summation on \( j \)); \( D_j(z, p) \) is the spatial diffusion coefficient; \( \sigma_j \) is the total cross section of nuclear fragmentation in the interstellar gas with mass density \( \rho(z) \) and the mean atomic mass \( m \); \( v \) is the particle velocity; reacceleration is described as diffusion in momentum space and is determined by the coefficient \( K_j(z, p) \); the term \( (dp/dt)_{\text{ion}} < 0 \) is responsible for ionization energy losses; \( q_j(z, p) \) is the source term; and

\[
S_{jk}(z, p) = \frac{\rho(z)}{m} \int_0^\infty dp' \, p'^2 v' \, \frac{\sigma_{jk}(p, p')}{dp'} \, f_j(z, p')
\]  

(2)

is the yield from more heavy nuclei which fragments into the \( j \)-type nuclei with production cross section \( \sigma_{jk}(p, p') \). For the sake of simplicity, the first term in equation (1) is frequently replaced by the term \( f_j/T_e \), where \( T_e \) is the characteristic escape time of cosmic rays from the Galaxy. This approach, the so-called leaky box approximation, offers a good fit to a great body of data on cosmic rays (Simpson 1983; Wefel 1988).

The leaky box approximation is theoretically justified in the case of the Galaxy with fast cosmic-ray diffusion and reflecting boundaries, where particles have some small probability of passing through (Ginzburg & Syrovatskii 1964). This situation is realized in a few Galactic models where a strong cosmic-ray streaming instability creates a barely transparent zone at some height above the Galactic plane (Ptuskin & Zirakashvili 1993), but we shall not discuss it here.

Another case in which the leaky box approximation may be applied to stable cosmic-ray nuclei in the Galactic disk is the flat halo model with the source and gas distributions concentrated in a disk which is much thinner than the halo (Ginzburg & Ptuskin 1976; Ptuskin & Soutoul 1990). Free exit of cosmic rays at the halo boundary is assumed in this model. It is easy to incorporate the process of reacceleration into this model provided that reacceleration occurs only in the region which is thin compared with the size of the halo, which strictly speaking is not the case, since reacceleration acts everywhere that scattering and spatial diffusion exist. Nevertheless, the rate of reacceleration may be higher near the Galactic plane, where the potential sources of interstellar turbulence (the supernova remnants and stellar winds) are distributed.

Under the assumption that the sizes of the gas disk \( h_g \), the source disk \( h_s \), the supernova remnants and stellar winds are distributed.

Now equation (1) has a simple solution in the halo, \( 0 < |z| \leq H \):

\[
f_j(z, p) = f_0(z) \left( 1 - \frac{|z|}{H} \right)^N,
\]  

(4)

where \( f_0 \) is the distribution function at the bottom of the halo and we assume for simplicity that \( D_j(z) = \text{constant} \).
wavenumber $k$:

$$W_k = \frac{wB^2L}{4\pi(1 - a)}(kL)^{-3+a}, \quad kL \geq 1, \quad a = \text{constant}.$$  \hfill (8)

Here $w = \langle f_{\nu,\nu} W_k / (B^2/4\pi) \rangle$ characterizes the level of turbulence, and $L$ is the principal scale of the turbulence.

The spatial and momentum diffusion coefficients are determined by the expressions (see Berezinskii et al. 1990)

$$D = \frac{v^2}{6} \int_0^1 \frac{1 - \mu^2}{\nu} \left( \frac{2}{1 - a} \right) \frac{vL}{\nu} \left( \frac{r_e}{L} \right)^a,$$  \hfill (9)

and

$$K_0 = \frac{2p^3}{v^2} \frac{dAx}{v} \int_0^1 \frac{r_e(1 - \mu^2)^\nu}{\nu} \frac{2\pi}{(1 - a)(2 - a)(4 - a)} \left( \frac{r_e}{L} \right)^a,$$  \hfill (10)

where $r_e$ is the particle gyroradius, $\mu$ is the cosine of the pitch angle, $v_A$ is the Alfvén velocity, and the frequency of particle scattering $\nu$ is determined by the wave energy density at the resonant wavenumber $k_{r,s} = 1/|\mu| r_e$:

$$\nu = \frac{2\pi}{v^2} \frac{k_{r,s} W_{r,s}}{r_e B^2}.$$  \hfill (11)

Cosmic-ray particles with energies less than $10^{15} - 10^{17}$ eV are strongly magnetized (i.e., the particle gyroradius is small compared with the diffusion mean free path along the magnetic field), and they diffuse predominantly along the local magnetic field. In the derivation of equation (9) we assumed that (due to the large-scale wandering of magnetic field lines) the average diffusion coefficient $D$ is a factor of 3 smaller than the local diffusion coefficient along the magnetic field line.

It is convenient now to write equation (5) in a different form. Instead of the distribution function $f_0(p)$, we shall use the cosmic-ray intensity $I(E) = A_p p^2 f_0(p)$ [measured in the units particles (cm$^{-2}$s sr eV nucleon$^{-1}$)$^{-1}$] as a function of kinetic energy per nucleon ($E$), where $A_p$ is the atomic number. This will simplify the last term in equation (5), since the energy per nucleon is approximately conserved during the fragmentation.

With the use of equations (5), (6), (9), and (10) we obtain the following equation for the cosmic-ray intensity $I(E)$ at the Galactic disk

$$I_j = \frac{I_j}{X_e} + \frac{\sigma_j}{m} I_j + \alpha \left[ \frac{A_j (E + E_0)}{Z_j} \frac{dX_e}{dE} + X_e \right] I_j$$

$$- \frac{A_j}{2Z_j} \frac{\beta(E + E_0)^2}{\beta} \frac{dX_e}{dE} \frac{dI_j}{dE} - \frac{\beta^2}{2} (E + E_0)^2 X_e \frac{d^2I_j}{dE^2}$$

$$+ \frac{d}{dE} \left( \frac{dE}{dx} \right)_{j,ion} I_j = \frac{Q_j}{\mu_0} + \sum_{k<j} \frac{\sigma_k}{m} I_k.$$  \hfill (12)

Here the parameter $\alpha$, which is defined as

$$\alpha = \frac{8}{3\alpha(4 - a)^2(4 - a)} \frac{h_\nu}{(v_e^2 - \nu) (\mu_0 h_e c)^2}.$$  \hfill (13)

[its dimension is (g cm$^{-2}$)] determines the efficiency of reacceleration; ionization energy losses per unit matter thickness are described by the term $(dE/dx)_{j,ion} < 0$; the source term $Q_j(E) = A_p p^2 q_0(p)$ gives cosmic-ray production in units of particles (cm$^{-2}$s sr eV nucleon$^{-1}$)$^{-1}$; $\beta = v/e$; and $E_0 = mc^2$ is
the nucleon rest mass energy. According to equations (6) and (9) we suppose that the escape length \( X_e \) in equation (12) is a function of particle magnetic rigidity \( R = \frac{pc}{Ze} \) and that it has a single-power-law form

\[
X_e(R) = X_0 R^{-\alpha},
\]

where \( R \) is measured in gigavolts.

It is clear from equation (12) that the empirical application of our model of cosmic-ray transport involves two parameters: the rigidity-dependent escape length \( X_e(R) \) and the constant \( \alpha \), which is the parameter of reacceleration. The ratio of the characteristic time of particle escape from the Galaxy to the characteristic time of particle acceleration is order of \( \alpha X_e^2 \). In principle, equation (12) is valid for any strength of acceleration, but below in our numerical calculation only the weak acceleration case is considered. Their dependences on the characteristics of the interstellar turbulence and the gas distribution are given by equations (6), (9), and (13).

Our knowledge of the parameters of the interstellar medium is too uncertain, and our theoretical model is too simplified, for reliable calculations of the parameters \( X_e \) and \( \alpha \). Nevertheless, the form of the transport equation (12) is rather general and is determined by the natural assumption that the scattering on random hydromagnetic waves causes the spatial and energy diffusion of cosmic-ray particles.

The values of \( X_e(R) \) and \( \alpha \) will be found in the next section from fits to the cosmic-ray data. Here we present order-of-magnitude estimates based on the relevant astrophysical information about the interstellar medium.

We suppose that the interstellar turbulence has the Kolmogorov spectrum with \( a = \frac{1}{2} \), and the wave energy density is equal to the regular magnetic field energy density at the principal scale, i.e., \( w = 1 \) (Armstrong, Cordes, & Rickett 1981). Accepting the value of the surface gas density \( \rho_g h = 10^{-7} \) g cm\(^{-2}\) and with parameters of the halo \( H = 3 \) kpc, \( B = 10^{-6} \) G, and \( L = 500 \) pc, we obtain from equation (9) an estimate for the escape length \( X_e = 15 (R/1 \text{ GV})^{-1/3} \) g cm\(^{-2}\). The reacceleration parameter determined from equation (13) is equal to \( \alpha = 10^{-3} (\text{g cm}^{-2})^{-2} \) for the Alfvén velocity \( v_A = 2 \times 10^6 \) cm s\(^{-1}\), which is typical of the warm ionized phase of the interstellar gas at the Galactic plane, and the size of the reacceleration region \( h = 1 \) kpc.

3. PROPAGATION CALCULATIONS

Using a weighted-slab propagation technique, equation (12) was solved with a propagation code that was initially developed for the standard leaky box model calculation. The prototype of this code was originated by Comstock (1969) and subsequently revised by Protheroe and Ormes (Protheroe, Ormes, & Comstock 1981) and Webber (1993). The latest version of Webber's code has been revised by us to add the reacceleration effect represented by the third term of equation (12). This code includes the effects of ionization energy loss, total inelastic cross sections, fragmentation partial cross sections, electron capture and loss cross sections, radioactive decays, composition of the interstellar medium, and the cosmic-ray source abundances and source energy spectra. Starting with a set of relative abundances of the elements and isotopes at the source and assuming a common source energy spectrum, 78 isotopic "species" ranging from \(^1\text{H}\) through \(^{62}\text{Ni}\) are propagated through slabs of interstellar matter and integrated over an exponential distribution of path lengths having the mean value \( X_e \). The code now computes the local interstellar spectra for an escape length \( X_e \) and the reacceleration parameter \( \alpha \).

For the escape length, a pure power law in rigidity, \( X_e = X_0 (R/1 \text{ GV})^{-1/3} \) g cm\(^{-2}\), which agrees with the Kolmogorov-type spectrum of hydromagnetic waves, was assumed. For the source spectra \( Q_j \), a power law in rigidity with the spectral index of 2.4, was assumed for each species. From the estimates in the previous section, the reacceleration parameter \( \alpha = a_0 \times 10^{-3} \) (g cm\(^{-2}\))\(^{-2}\) was assumed. The constants \( X_0 \) and \( a_0 \) were adjusted to get good agreement between the calculated and observed B/C ratios. The calculated H and He spectra were also required to agree with the observed spectra. For the chosen \( X_0 \) and \( a_0 \) parameter set, calculations of the ratios among the H and He isotopes, the Fe spectrum, and the sub-Fe to Fe ratio are also compared with the observed data in this paper. In this calculation, the instellar medium was assumed to be 90% hydrogen and 10% helium, and the additional energy loss due to ionized hydrogen was included (Soutoul, Ferrando, & Webber 1990).

For each species, the local interstellar spectrum obtained from the Galactic propagation calculation was subjected to solar modulation based on a numerical solution for a spherically symmetric model (Fisk, Forman, & Axford 1973) including diffusion, convection, and adiabatic deceleration. A solar wind speed \( v = 400 \) km s\(^{-1}\) and a heliospheric boundary distance \( r_0 = 80 \) AU were assumed for these modulation calculations. We have used the diffusion coefficient \( k(R) = k_0 R^\beta \), where \( R \) is the rigidity, \( \beta \) is the velocity of the cosmic rays, and \( k_0 \) is a constant that can be adjusted to get the best agreement between the calculated and observed spectra. With this form of the diffusion coefficient the solar modulation level at radius \( r_0 \) in the heliosphere can be characterized by the parameter \( \phi = v(r_g - r_0)/3k_0 \). The modulated spectra with the solar modulation parameter \( \phi = 500 \) MV were compared with the compiled data, which were measured mostly at times of solar minimum modulation.

For the B/C ratio, combinations of \( X_0 = 13–15 \) and \( a_0 = 1–2 \) produced reasonably good fits to the data. Note that Heinbach & Simon (1993) also found a similar parameter set. The combination of \( X_0 = 14 \) and \( a_0 = 1 \) gave the best simultaneous fit to both the B/C ratio and the light-element (H and He) data. This is just a first-order choice, and the parameters are not yet perfectly tuned to fit all the data. However, as shown in the following section, the use of this combination in our calculation agrees well with the compiled data.

4. RESULTS AND DISCUSSIONS

4.1. B/C Ratio

The B/C ratio is the best-measured secondary-to-primary ratio, and it has been used by many authors, both to investigate the cosmic-ray path length distribution (Garcia-Munoz et al. 1987) and to tune the escape length values for all secondary-to-primary ratios (Ferrando et al. 1991). Other secondary-to-primary ratios, e.g., Li/C, Be/C, and F/Ne, could be employed in place of the B/C ratio, but none of the former have been measured in cosmic rays as well as the B/C ratio. In general, the usefulness of ratios other than B/C is limited by the relatively low accuracy of their measurements or by the poor knowledge of the cross sections for the secondary species production. Not only has the B/C ratio been rather well measured in cosmic rays, but the cross sections for the production of their secondaries are also relatively well known.
Figure 3 shows the agreement between our calculated B/C ratio and the compiled data. The solid curve represents the calculated results, and the symbols represent the compiled data (Garcia-Munoz et al. 1987 and references therein). The stochastic reacceleration reproduces the peak in the B/C ratio around 1 GeV nucleon\(^{-1}\) without any ad hoc escape length. The standard leaky box model is generally fitted to the B/C ratio by adjusting the escape length to be \(\beta\)-dependent, i.e., a rigidity power law with index \(\delta\) above a threshold rigidity \(R_0\) and proportional to \(\beta^\alpha\) below \(R_0\); specifically, \(X_e = X_0 \beta^\alpha R^{-\delta}\) for \(R \geq R_0\) GV and \(X_e \sim \beta^\alpha\) for \(< R < R_0\) GV. For example, commonly used escape length types are \(X_e = 35.1 \beta^{-0.6}\) for \(R \geq 3.3\) GV and \(X_e = 17.2 \beta\) for \(< R < 3.3\) GV (Webber et al. 1992); \(X_e = 25.4 \beta^{-0.59}\) for \(R \geq 4.2\) GV and \(X_e = 11.1 \beta\) for \(< R < 4.2\) GV (Webber 1993); \(X_e = 35.7 \beta^{-0.65}\) for \(R \geq 4.35\) GV and \(X_e = 14.4 \beta\) for \(< R < 4.35\) GV (Ferrando et al. 1991); and \(X_e = 42.4 \beta^{0.23} R^{-0.65}\) for \(R \geq 3.7\) GV and \(X_e = 18.1 \beta^{0.12}\) for \(< R < 3.7\) GV (Gibner et al. 1992). The parameters \(\delta, \beta, R_0,\) and \(\gamma\) are chosen ad hoc, and they have little or no theoretical justification (Jones 1979; Berezinskii et al. 1990). They have been introduced only to provide closer agreement between the model calculation and the observed data, i.e., they are introduced specifically for fitting purposes. However, in the reacceleration model, the peak of the B/C ratio around 1 GeV nucleon\(^{-1}\) can be explained as a reacceleration effect with a pure rigidity power-law escape length, \(X_e = 14(\text{GV})^{1/3}\) g cm\(^{-2}\), which agrees with the Kolmogorov-type spectrum of hydromagnetic waves.

4.2. H and He Spectra

Cosmic-ray confinement in the Galaxy is energy-dependent, and the higher energy particles escape more freely than the low-energy ones. Therefore, it is expected that reacceleration could modify the spectral shape of cosmic rays in the energy region where they spend more time in the Galaxy. In the case of heavy nuclei, the interaction loss dominates, so their spectra may not reflect the effects of reacceleration. However, reacceleration effects should be especially noticeable for light elements, particularly H and He, whose interaction mean free paths are much longer than their escape mean free paths (Stephens & Golden 1989). Stephens & Golden (1989) have studied the effect of reacceleration on the spectral shape of H and He and concluded that reacceleration provides a poor fit to the observed data. It was also pointed out by Cesarsky (1987) that the H source spectrum could be expected to develop an unwanted bump in the \(~1\) GeV region when reacceleration is introduced, whereas it would tend to "iron out" the heavy primary source spectra.

In any case, it is important to check the light-element spectra calculated with the reacceleration model, since the reacceleration effects are stronger for the lighter elements. Our calculated spectra for the quartet of H and He isotopes obtained with the parameter set \(X_e = 14(\text{GV})^{1/3}\) g cm\(^{-2}\) and \(\alpha = 1 \times 10^{-3}\) (g cm\(^{-2}\))\(^{-2}\), which gives a good fit to the B/C ratio, are compared with the compiled data over a wide energy range in Figures 4 and 5. For more effective reacceleration, i.e., for a larger \(\alpha\), the \(^{1}\text{H}\) spectrum would develop a larger bump in the \(~1\) GeV region. However, the amount of reacceleration that was sufficient to reproduce the B/C ratio in our calculation seems to have no appreciable bump structure in the H spectrum, i.e., it causes no significant discrepancy between the calculated spectra and the compiled data.

In fact, our calculated spectra agree well with the compiled data over the whole energy range where the measurements are available for the quartet of H and He isotopes. By contrast, the standard leaky box model with the example escape lengths mentioned in § 4.1 could not fit the primary H and He data over this wide energy range. In the leaky box model, the high-energy spectrum of a pure primary species is given by the simple relation \(J = Q \Lambda\), where \(Q\) is the source spectrum and \(\Lambda = 1/(\nu_{\text{escape}} + 1/\nu_{\text{reacceleration}})\) is the mean path length. The rigidity power law \(\Lambda\) with the index of 0.6 applied to the rigidity power-law source spectra with spectral indices of 2.3 or 2.36, which were preferentially used to interpret the low-energy data (Webber et al. 1992; Lukasiak et al. 1994), result in excessively steep high-energy spectral indices (2.9 = 0.6 + 2.3, 2.96 = 0.6 + 2.36). The flatter source spectra (power law with index 2.1) can give reasonable high-energy spectral shape (2.7 = 2.1 + 0.6), but it results in low-energy fluxes that are too small when they are normalized to the high-energy data. Nevertheless, both the H spectrum and the H/He ratio of \(~1\) GeV nucleon\(^{-1}\) are sensitive to reacceleration effects, so it is important to have accurate measurements of H and He over a wide energy range.

The calculated H/He ratio is also compared with the compiled data in Figure 6. Our calculated ratio does not show the strong feature found by Stephens & Golden (1990), which is incompatible with the data. Using the calculations of Stephens & Golden (1990), which were based on the redistributed reacceleration model by Wandel, Seo et al. (1991) concluded that the H/He ratio data do not support reacceleration of the magnitude suggested for the heavier nuclei by Wandel. Although our current recalculation was not originally intended to tune the H/He ratio, it does show a reasonable fit to the ratio data over a wide energy range. The detailed structure of the ratio could be a good signature for the reacceleration effect, and it can give constraints on the amount of reacceleration. For more effective reacceleration, i.e., for a larger \(\alpha\), the H/He ratio would show a larger bump around \(~1\) GeV nucleon\(^{-1}\). However, the small reacceleration that reproduced the B/C ratio does not produce a bump that is incompatible with the compiled H/He ratio data.

The highest energy (> 2 TeV nucleon\(^{-1}\)) data from the JACEE collaboration (Burnett et al. 1990) show a factor of 2
lower H/He ratio than would be expected from an extrapolation of the lower energy (less than 100 GeV nucleon\(^{-1}\)) data of Ryan, Ormes, & Balasubramanyan (1972). With our reacceleration calculation, the discrepancy between the predicted ratio and the JACEE data is not as significant as the discrepancy addressed earlier (Burnett et al. 1990) based on the standard leaky box model prediction. The lower ratio reported by JACEE seems to be due to the fact that the measured He flux is higher than the calculated one. Our reacceleration model produces a slightly lower He flux than the JACEE He data, while it agrees well with the JACEE H data. Therefore, a slightly flatter source spectrum for He than for H might explain the discrepancy. However, in this paper we do not focus on a detailed fit to individual sets of the data, but instead focus on a first-order test of the reacceleration effect. There is a special need for accurate measurements of the H/He ratio in the energy range from 100 GeV nucleon\(^{-1}\) to 1 TeV nucleon\(^{-1}\), where no measurements have yet been made.

It is also interesting to check the effects of reacceleration on the H and He isotopes. In Figure 7, our calculated ratios among the H and He isotopes are compared with both the most recent measurements of the Voyager 2 spacecraft (Seo et al. 1993) and the standard leaky box model calculations with different escape length types. In this case, the calculated curves were obtained with the solar modulation parameter \(\phi = 360\) MV, since the data were obtained from 23 AU during the 1987 solar minimum (Seo et al. 1993). The solid curves represent our reacceleration calculation with \(X_e = 14(R/1 \text{ GV})^{-1/3} \text{ g cm}^{-2}\) and the reacceleration parameter, \(\alpha = 1 \times 10^{-3} \text{ (g cm}^{-2}\)^{-2}\). The dotted curves represent the standard leaky box model calculation with \(X_e = 35.1R^{-0.6} \text{ g cm}^{-2}\) for \(R \geq 3.3 \text{ GV}\) and \(X_e = 17.2 \beta g \text{ cm}^{-2}\) for \(R < 3.3 \text{ GV}\) (Webber et al. 1992). The long-dashed curves represent the standard leaky box model calculation with \(X_e = 35.1R^{-0.6} \text{ g cm}^{-2}\) for all rigidities (Webber et al. 1992). The short-dashed curves represent the standard leaky box model calculation with \(X_e = 8(5.5/R)^{0.6} \text{ g cm}^{-2}\) for \(R \geq 5.5 \text{ GV}\) and \(X_e = 8 \text{ g cm}^{-2}\) for \(R < 5.5 \text{ GV}\) (Stephens 1989). Our calculated ratios with reacceleration show agreement with the data that is as good as the standard leaky box model calculations. Without reacceleration, the
FIG. 5.—Comparison of the spectra (solid curve) calculated using our reacceleration model with the compiled data for (a) the $^4$He and (b) the $^3$He spectra. The legends for the symbols are the same as in Fig. 4, with one additional data set: dotted open circles: Buckley et al. (1993).

FIG. 6.—Comparison of the ratio (solid curve) calculated using our reacceleration model with the compiled data for the H/He ratio. Open squares: (IMP 8) Reames 1990; filled circles: Seo et al. (1991); open diamonds: Webber & Yushak (1983); upward pointing open triangles: Rygg & Earl (1971); downward pointing open triangles: Webber et al. (1987); open circles: Simpson (1983); filled squares: Ryan et al. (1972); filled diamonds: the JACEE data (Burnett et al. 1990).
also wipes out the $^3$H production cross section shape in the spectrum, accurate $^3$H measurements from a few hundred MeV nucleon$^{-1}$ to a few GeV nucleon$^{-1}$ at solar minimum would be a good test for the reacceleration model.

4.3. Fe Spectrum and Sub-Fe to Fe Ratio

Although the heavy primary spectra are not expected to exhibit the effects of reacceleration as readily as the light primary spectra, it is worthwhile to check whether we can predict a reasonable spectrum for a heavy primary with the same parameters used to reproduce the B/C ratio and the H and He spectra. The calculated Fe spectrum, which is compared with the compiled data in Figure 8, does not show any significant reacceleration effect. The calculated spectrum was obtained with the same parameter set, $X_e = 14(R/1 \text{ GV})^{-1/3} \text{ g cm}^{-2}$ and $\alpha = 1 \times 10^{-5} \text{ (g cm}^{-2})^{-2}$, which gave a good fit to both the B/C ratio and the H and He data simultaneously.

Since the B/C ratio is predominantly sensitive to long path lengths and the sub-Fe to Fe ratio is predominantly sensitive to short path lengths in the path-length distribution, these two ratios have been useful tools for studying the cosmic-ray path-length distribution (Garcia-Munoz et al. 1987). The evidence that it is difficult to fit these two ratios simultaneously, if the path-length distribution is exponential, has been accumulating over the years. The calculated sub-Fe to Fe ratios based on a standard leaky box model lie consistently below the measurements. In order to reproduce simultaneously the measured B/C and sub-Fe to Fe ratios, it has been suggested that the path-length distribution is not purely exponential but is depleted in short path lengths, perhaps even in an energy-dependent way (Garcia-Munoz et al. 1987).

To check the effects of reacceleration on the apparent need for truncation of short path lengths, the calculated sub-Fe to Fe ratio with the same parameter set, $X_e = 14(R/1 \text{ GV})^{-1/3} \text{ g cm}^{-2}$ and $\alpha = 1 \times 10^{-5} \text{ (g cm}^{-2})^{-2}$, which reproduced the B/C ratio and the H and He data is compared with the compiled data in Figure 9. At low energies our calculated ratio is slightly higher than the standard leaky box model calculation by Garcia-Munoz et al. (1987). However, the discrepancy with the data still exists, and it seems that a truncation of a similar (or slightly smaller) amount of the standard leaky box model may still be needed. The truncation calculation of Ptuskin & Soutoul (1990) in their cloudy interstellar medium model predicts a small underproduction of light secondaries such as $^3$He as compared to the standard leaky box model, and it tends to improve the fits for the light isotopes discussed in § 4.2.

5. CONCLUSIONS

The effects of reacceleration on cosmic rays have been studied over a wide charge and energy range using a model of reacceleration by the interstellar turbulence suggested by Osborne & Ptuskin (1988) and recently considered by Heinbach & Simon (1993). The calculations agree well with the observed data for the light elements, H and He, as well as for the medium elements, the B/C ratio, and the heavy elements (Fe). In this model, the data can be fitted well with a simple rigidity power-law escape length corresponding to the Kolmogorov-type spectrum of the interstellar turbulence. No further modifications of the confinement mechanism, which is required for the standard leaky box model below a few GeV nucleon$^{-1}$, have to be invoked. The escape length $X_e = 14(R/1 \text{ GV})^{-1/3} \text{ g cm}^{-2}$ and the reacceleration parameter (Seo et al. 1993) is reflected in the $^3$H-related ratios as well as in the $^3$H spectrum. The strong decreases of the $^3$H-related ratios slightly above 100 MeV nucleon$^{-1}$ illustrate this effect. With reacceleration, however, the particles are redistributed and, therefore, smooth out the cross section shape. Since solar modulation energy dependence of $^3$H production cross section (Seo et al. 1993) is reflected in the $^3$H-related ratios as well as in the $^3$H spectrum.
\[ \alpha = 1 \times 10^{-3} \left( \text{g cm}^{-2} \right)^{-2} \]
determined from the data in § 3 are in good agreement with the theoretical estimates given in § 2.

The lighter species are more sensitive to the effects of reacceleration, and only a small amount of reacceleration is allowed in fits to the light-element data. The H and He data put stronger constraints than the B/C data on the reacceleration parameter \( \alpha \), and it seems that \( \alpha \) does not exceed the chosen value by more than about a factor of 2, i.e., \( \alpha \lesssim 2 \times 10^{-3} \left( \text{g cm}^{-2} \right)^{-2} \).

Measurements of the H/He ratio over a wide energy range during the solar minimum can provide a good test for this model. It is also important to extend the measurements of \(^{2}\text{H}\) and \(^{3}\text{He}\) isotopes to higher energies, where the solar modulation effects are less and the \(^{2}\text{H}\) and \(^{3}\text{He}\) data are more directly comparable to the more precise B/C and sub-Fe to Fe ratio data available above \( \sim 700 \text{ GeV nucleon}^{-1} \) (Engelmann et al. 1990).

Our calculated sub-Fe to Fe ratio is similar to calculations with the standard leaky box model. The reacceleration effect does not remove the need for truncation of short path lengths. Our calculations have shown that the cosmic-ray data can be represented by a reacceleration model with a simple rigidity power-law escape length at least as well as they can by the standard leaky box model with its various types of ad hoc escape lengths.

**Fig. 8.—**Comparison of the spectrum calculated using our reacceleration model with the compiled data for the Fe spectrum. Filled circles: Engelmann et al. (1990); downward pointing filled triangles: the JACEE data (Asakamori et al. 1993); filled squares: Muller et al. (1991); open circles: Ferrando et al. (1991); open squares: Juliussen (1974); open diamonds: Sato et al. (1985); upward pointing filled triangles: Benegas et al. (1975); filled diamonds: Minagawa et al. (1981); dotted open circles: Simon et al. (1980); dotted open squares: Orth et al. (1978); upward pointing open triangles: Chappell & Webber (1981).

**Fig. 9.—**Comparison of the ratio calculated using our reacceleration model with the compiled data for the sub-Fe to Fe ratio (Garcia-Munoz et al. 1987 and references therein).
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